Exercise 11

(a) A company makes computer chips from square wafers of silicon. It wants to keep the side length of a wafer very close to 15 mm and it wants to know how the area $A(x)$ of a wafer changes when the side length x changes. Find $A'(15)$ and explain its meaning in this situation.

(b) Show that the rate of change of the area of a square with respect to its side length is half its perimeter. Try to explain geometrically why this is true by drawing a square whose side length x is increased by an amount Δx. How can you approximate the resulting change in area ΔA if Δx is small?

Solution

Part (a)

Take the derivative of

$$A(x) = x^2$$

to find how the area changes when the side length changes.

$$A'(x) = 2x$$

Consequently,

$$A'(15) = 2(15) = 30 \text{ mm}.$$

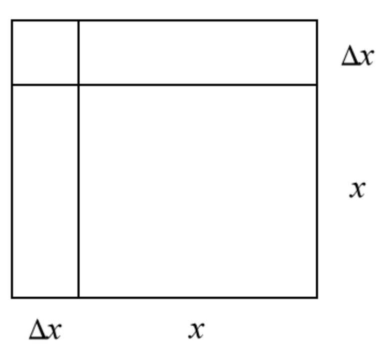
This means that when the side length is 15 mm, the area is increasing by 30 mm^2 per millimeter of side length.

Part (b)

The perimeter of a square is $P = x + x + x + x = 4x$, so

$$A'(x) = \frac{P}{2}.$$

Suppose there's a square with side length x, and the side length then increases by Δx.



The old area is $A_{\text{old}} = x^2$, and the new area is

$$A_{\text{new}} = x^2 + x\Delta x + x\Delta x + \Delta x\Delta x$$

$$= x^2 + 2x\Delta x + (\Delta x)^2.$$

Because Δx is assumed to be small, $(\Delta x)^2$ is extremely small compared to $x^2 + 2x\Delta x$ and can be neglected to a good approximation.

$$A_{\text{new}} \approx x^2 + 2x\Delta x$$

Therefore, the approximate change in area is

$$\begin{aligned} \Delta A &= A_{\text{new}} - A_{\text{old}} \\ &\approx (x^2 + 2x\Delta x) - x^2 \\ &\approx 2x\Delta x. \end{aligned}$$